Chapter 1

Exercise Solutions

1. a) $\frac{1 \text{ in}}{2.54 \text{ cm}}$ or $\frac{2.54 \text{ cm}}{1 \text{ in}}$

   b) $\frac{1.6 \text{ km}}{1 \text{ mi}}$ or $\frac{1 \text{ mi}}{1.6 \text{ km}}$

   c) $\frac{60 \text{ arcmin}}{3600 \text{ arcsec}}$ or $\frac{3600 \text{ arcsec}}{60 \text{ arcmin}}$

2. a) $12 \cancel{\text{in.}} \times \frac{2.54 \text{ cm}}{1 \cancel{\text{in.}}} = 12 \times 2.54 \text{ cm} = 30.5 \text{ cm}$

   b) $100 \cancel{\text{cm}} \times \frac{1 \text{ in.}}{2.54 \cancel{\text{cm}}} = \frac{100}{2.54}\text{in.} = 39 \text{ in.}$

   c) $380,000 \cancel{\text{km}} \times \frac{1 \text{ mi.}}{1.6 \cancel{\text{km}}} = \frac{380{,}000}{1.6}\text{mi.} = 240,000 \text{ mi.}$

   d) $93,000,000 \cancel{\text{mi.}} \times \frac{1.6 \text{ km}}{1 \cancel{\text{mi.}}} = 150,000,000 \text{ km}$

   e) $0.5 \cancel{\text{deg}} \times \frac{3600 \text{ arcsec}}{1 \cancel{\text{deg}}} = 1,800 \text{ arcsec}$

3. a) A 1-foot ruler marked with both inches and cm has exactly 12 inches and just over 30 cm.

   b) 100 cm is about 3 times the previous answer, and 39 inches is about 3 times the previous one also.

   c) The number of miles, 240,000, is a little smaller than the number of kilometers, 380,000. This makes sense becuase the distance uni miles is slightly larger than the distance unit kilometers (1 mile is equivalent to 1.5 kilometers), so there should be slightly fewer miles to express the same distance as there are kilometers.

d) This is the same as the previous part (c) explanation, but reversed. Since the distance unit kilometers is slightly smaller than the distance unit miles (becaue it takes 1.6 km to make up just 1 mi), there should be slightly more kilometers than miles to express the same distance. And just as the conversion factor shows, it is not a lot more (1.6 compared to 1), ut a comparable amount. So it makes sense that both answers are in the millions. It is only about half again as many km as miles.

e) Drawing directly from the conversion factor, or equivalence relation deg $\leftrightarrow$ 3600 arcsec, half as many degrees (0.5) should equal half as many arcseconds (1800).

4. a) $60\frac{\cancel{\text{mi.}}}{\cancel{\text{hr}}} \times \frac{1.6\ \cancel{\text{km}}}{1\ \cancel{\text{mi}}} \times \frac{1000\ \cancel{\text{m}}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{hr}}}{3600\ \text{sec}} = \frac{60\times1.6\times1000}{3600}\frac{\text{m}}{\text{sec}} = 27\ \text{m/s}$

b) $1\ \cancel{\text{day}} \times \frac{24\ \cancel{\text{hrs}}}{1\ \cancel{\text{day}}} \times \frac{3600\ \text{sec}}{1\ \cancel{\text{hr}}} = 86,400\ \text{sec}$

or, splitting the hours $\rightarrow$ seconds conversion,

$1\ \cancel{\text{day}} \times \frac{24\ \cancel{\text{hrs}}}{1\ \cancel{\text{day}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}} \times \frac{60\ \text{sec}}{1\ \cancel{\text{min}}} = 86,400\ \text{sec (same answer)}$

c) Assume 1 dollar since no other number was given:

$\frac{1\ \cancel{\text{dollar}}}{\cancel{\text{kg}}} \times \frac{100\ \text{cents}}{1\ \cancel{\text{dollar}}} \times \frac{1\ \cancel{\text{kg}}}{1000\ \text{g}} = 0.1\frac{\text{cents}}{\text{g}}$

d) $1\ \cancel{\text{mile}} \times \frac{1760\ \cancel{\text{yd}}}{1\ \cancel{\text{mile}}} \times \frac{3\ \cancel{\text{ft}}}{1\ \cancel{\text{yd}}} \times \frac{12\ \cancel{\text{in.}}}{1\ \cancel{\text{ft}}} \times \frac{1\ \text{step}}{30\ \cancel{\text{in.}}} = \frac{1760\times3\times12}{30}\ \text{steps} = 2100\ \text{steps}$

5. a) $1\ \cancel{\text{in}^2} \times \left(\frac{1\ \text{ft}}{12\ \cancel{\text{in}}}\right)^2 = \frac{1\ \text{ft}^2}{12^2} = \frac{1}{144}\ \text{ft}^2 = 0.0069\ \text{ft}^2$

b) $1\ \cancel{\text{ft}^3} \times \left(\frac{12\ \text{in}}{1\ \cancel{\text{ft}}}\right)^3 = 12^3\ \text{in}^3 = 1730\ \text{in}^3$

c) $1\ \cancel{\text{m}^2} \times \left(\frac{100\ \text{cm}}{1\ \cancel{\text{m}}}\right)^2 = 100^2\ \text{cm}^2 = 10,000\ \text{cm}^2$

d) $1\ \cancel{\text{yd}^3} \times \left(\frac{3\ \text{ft}}{1\ \cancel{\text{yd}}}\right)^3 = 3^3\ \text{ft}^3 = 27\ \text{ft}^3$

6. a) $1\frac{\cancel{\text{N}}}{\text{m}^2} \times \left(\frac{1\frac{\text{kg m}}{\text{s}^2}}{1\,\cancel{\text{N}}}\right) = \frac{1\text{ kg}\,\cancel{\text{m}}}{\text{s}^2} \times \frac{1}{\text{m}\cancel{^2}} = \frac{1\text{ kg}}{\text{m s}^2}$

   b) $1\frac{\cancel{\text{J}}}{\text{m}^3} \times \left(\frac{1\cancel{\text{N}}\,\text{m}}{1\,\cancel{\text{J}}}\right) \times \left(\frac{1\frac{\text{kg m}}{\text{s}^2}}{1\,\cancel{\text{N}}}\right) = \frac{1\,\cancel{\text{m}}\frac{\text{kg}\,\cancel{\text{m}}}{\text{s}^2}}{\text{m}\cancel{^3}} = \frac{1\text{ kg}}{\text{m s}^2}$

   Note that this is the same answer as (a). Thus the dimensions of pressure (force per area) are equivalent to those of energy density (energy per volume).

7. R = 6371 km (the average radius of the Earth) for all exercises here:

   a) $C = 2\pi R = 2\pi(6371\text{ km}) = 40{,}000\text{ km}$

   b) $SA = 4\pi R^2 = 4\pi(6371\text{ km})^2 = 510{,}000{,}000\text{ km}^2$

   c) $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(6371\text{ km})^3 = 1{,}080{,}000{,}000{,}000\text{ km}^3$
   or 1.08×10^{12} km^3 (scientific notation is covered in Section 1.4)

8. a) subtraction: 452 m - 1.7 m = 450.3 m. Therefore the building is 450.3 meters taller than a person.

   division: (452 m)/(1.7 m) = 266. Therefore the building is 266 times taller than a person.

   Division is more useful in this case because the umbers are not comparable in size. If you don't have a sense of how big 452 meters is, the result of subtraction (450.3 m) doesn't help give you that sense. However, saying that the building is 266 times larger does help give you that sense, because you can try to picture a couple hundred people stacked head to toe.

   b) subtraction: 220 lbs - 195 lbs = 25 lbs. The man lost 25 pounds, or he used to weigh 25 lbs more.

   division: (220 lbs)/(195 lbs) = 1.13 (or 113%).

Both methods are useful here. If you are familiar with the unit pounds for measuring weight, then you have a sense that 25 pounds is a substantial amount of weight – equivalent to about 3 gallons (11 liters) of water at Earth's surface, or the weight of a 2-year-old child. Subtraction helps you picture that amount of weight lost.

Division gives you a sense of how that lost weight compares to the man's total weight. It was 13% more, which is a substantial but not dominant fraction of his original weight.

c) subtraction: 200,000,000,000 stars - 400,000 stars = 199,999,600,000 more stars.

division: (200,000,000,000 stars) / (400,000 stars) = 500,000 (half a million) times more.

Division is more useful here. The two numbers are so vastly different (by 6 orders of magnitude) that subtraction gives you essentially the same number you started with.

9. $\mathrm{R}_{\mathrm{Sun}} = 109\mathrm{R}_{\mathrm{Earth}}$ for the following parts:

a) $C = 2\pi R$, from Exercise 1.7

$$\frac{C_{\text{Sun}}}{C_{\text{Earth}}} = \frac{\cancel{2\pi} R_{\text{Sun}}}{\cancel{2\pi} R_{\text{Earth}}} = \frac{109\, \cancel{R_{\text{Earth}}}}{\cancel{R_{\text{Earth}}}} = 109$$

Therefore $\frac{C_{\text{Sun}}}{C_{\text{Earth}}} = 109$, or $C_{\text{Sun}} = 109\, C_{\text{Earth}}$. (The Sun's circumference is 109 times larger than Earth's.)

b) $SA = 4\pi R^2$, from Exercise 1.7.

$$\frac{SA_{\text{Sun}}}{SA_{\text{Earth}}} = \frac{\cancel{4\pi} R^2_{\text{Sun}}}{\cancel{4\pi} R^2_{\text{Earth}}} = \left(\frac{R_{\text{Sun}}}{R_{\text{Earth}}}\right)^2 = \left(\frac{109\, \cancel{R_{\text{Earth}}}}{\cancel{R_{\text{Earth}}}}\right)^2 = 109^2 = 12,000$$

Therefore $\frac{SA_{\text{Sun}}}{SA_{\text{Earth}}} = 12,000$. (The Sun's surface area is 12,000 times larger than Earth's.)

c) $V = \frac{4}{3}\pi R^3$, from Section 1.2.3.

$$\frac{V_{\text{Sun}}}{V_{\text{Earth}}} = \frac{\cancel{\frac{4}{3}\pi} R_{\text{Sun}}^3}{\cancel{\frac{4}{3}\pi} R_{\text{Earth}}^3} = \left(\frac{R_{\text{Sun}}}{R_{\text{Earth}}}\right)^3 = \left(\frac{109\,\cancel{R_{\text{Earth}}}}{\cancel{R_{\text{Earth}}}}\right)^3 = 109^3 = 1,300,000$$

Therefore $\frac{V_{\text{Sun}}}{V_{\text{Earth}}} = 1,300,000$. (The Sun's volume is 1.3 million times larger than Earth's.)

10. a) $\frac{C_{\text{a}}}{C_{\text{b}}} = \frac{1}{109}$, so $C_{\text{a}} = \frac{1}{109} C_{\text{b}}$. The circumference of object "a" is 109 times smaller than (or 1/109th as large as) object "b", so "b" is bigger by 109 times.

OR $C_{\text{b}} = 109\, C_{\text{a}}$. The circumference of object "b" is 109 times larger than the circumference of object "a", so again "b" is bigger.

b) $\frac{SA_1}{SA_2} = \frac{11{,}900}{1}$, so $SA_1 = 11,900\, SA_2$. The surface area of object 1 is 11,900 times that of object 2. So object 1 is bigger.

c) $\frac{V_{\text{j}}}{V_{\text{k}}} = \frac{1}{1.3 \times 10^6}$, so $V_{\text{j}} = \frac{1}{1.3\times 10^6} V_{\text{k}}$ OR $V_{\text{k}} = 1.3 \times 10^6\, V_{\text{j}}$. The volume of object k is 1.3 million times that of object j. So object k is bigger.

11. $v \propto d$, so $v = H_0 d$ where H_0 is a constant of proportionality.

$d_{\text{z}} = 50$ Mpc
$d_{\text{y}} = 800$ Mpc

$$\frac{v_{\text{z}}}{v_{\text{y}}} = \frac{H_0\, d_{\text{z}}}{H_0\, d_{\text{y}}} = \frac{50 \text{ Mpc}}{800 \text{ Mpc}} = \frac{50}{800} = \frac{1}{16} = .0625$$

$\frac{v_{\text{z}}}{v_{\text{y}}} = \frac{1}{16}$, or $v_{\text{y}} = 16\, v_{\text{z}}$. Galaxy y is moving 16 times as fast as z.

12. $f = \frac{c}{\lambda}$, from this section.

$f_{\text{UV}} = 100\, f_{\text{IR}}$

Rearrange $f = \frac{c}{\lambda} \quad \rightarrow \quad \lambda = \frac{c}{f}$

$$\frac{\lambda_{\text{UV}}}{\lambda_{\text{IR}}} = \frac{\frac{c}{f_{\text{UV}}}}{\frac{c}{f_{\text{IR}}}} = \frac{\cancel{c}}{f_{\text{IR}}} \times \frac{f_{\text{UV}}}{\cancel{c}} = \frac{f_{\text{IR}}}{f_{\text{UV}}} = \frac{\cancel{f_{\text{IR}}}}{100\,\cancel{f_{\text{IR}}}} = \frac{1}{100}$$

So, $\frac{\lambda_{\text{UV}}}{\lambda_{\text{IR}}} = \frac{1}{100}$: The wavelength of infrared light is 100 times larger than that of ultraviolet.

13. $\text{F}_{\text{g}} \propto \frac{1}{R^2}$, or $\text{F}_{\text{g}} \propto \frac{z}{R^2}$, from this section, where z is an arbitrary proportionality constant and we are ignoring mass (only considering distance, R).

near moon distance = d_{near}
far moon distance = d_{far}; $\text{d}_{\text{far}} = 3\text{d}_{\text{near}}$

$$\frac{F_{\text{g, near}}}{F_{\text{g, far}}} = \frac{z/\text{d}^2_{\text{near}}}{z/\text{d}^2_{\text{far}}} = \frac{\cancel{z}}{\text{d}^2_{\text{near}}} \times \frac{\text{d}^2_{\text{far}}}{\cancel{z}} = \frac{\text{d}^2_{\text{far}}}{\text{d}^2_{\text{near}}} = \frac{(3\cancel{\text{d}_{\text{near}}})^2}{\cancel{\text{d}^2_{\text{near}}}} = 9.$$

So, $\frac{F_{\text{g, near}}}{F_{\text{g, far}}} = 9$, or $F_{\text{g, near}} = 9F_{\text{g, far}}$: The closer moon experiences 9 times stronger gravity than the farther moon.

14. Eq. 1.11: $\quad \text{time} = \frac{\text{distance}}{\text{speed}}$

speed = 100 km/h

distance = $70\ \cancel{\text{miles}} \times \left(\frac{1.6\text{ km}}{1\ \cancel{\text{mi}}}\right)$ =112 km
(A unit conversion gave distance in units that matched the"km" in "km/h" of speed units.)

$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{112\ \cancel{\text{km}}}{100\ \cancel{\text{km}}/\text{hr}} = 1.12\text{ hr}$

15. Eq. 1.9: $\quad \text{distance} = \text{speed} \times \text{time}$

speed = 29.8 km/s

time $= 1\ \cancel{\text{min}} \times \frac{60\text{ sec}}{1\ \cancel{\text{min}}} = 60$ sec
(A unit conversion brought the time units into agreement.)

distance = speed × time = $(29.8 \frac{\text{km}}{\cancel{\text{s}}}) \times (60\ \cancel{\text{s}}) = 1788$ km

Converting this answer to meters gives $1788\ \cancel{\text{km}} \times \frac{1000\text{ m}}{1\ \cancel{\text{km}}} = 1.8 \times 10^6$ m, or just under 2 million meters.

16. amount = 217 pages

time $= 8\ \cancel{\text{hours}} \times \frac{60\text{ min}}{1\ \cancel{\text{hour}}} = 480$ minutes

rate = ?

rate $= \frac{\text{amount}}{\text{time}} = \frac{217\text{ pages}}{480\text{ min}} = 0.45$ pages/min (or equivalently, 2.2 minutes per page, which is the inverse)

17. rate = 0.45 pages/min

amount = 350 pages

time $= \frac{\text{amount}}{\text{rate}}$ (Eq. 1.13)

Using the absolute method,
time $= \frac{350\ \cancel{\text{pages}}}{0.45\ \cancel{\text{pages}}/\text{min}} = 778$ minutes, or almost 13 hours.

Using the ratio method, which would be convenient if you had not already calculated the rate in problem 1.16,

$$\frac{\text{time}_{\text{long}}}{\text{time}_{\text{short}}} = \frac{\text{amount}_{\text{long}}/\text{rate}}{\text{amount}_{\text{short}}/\text{rate}} = \frac{\text{amount}_{\text{long}}}{\text{amount}_{\text{short}}}$$

$$\text{time}_{\text{long}} = \text{time}_{\text{short}} \left(\frac{\text{amount}_{\text{long}}}{\text{amount}_{\text{short}}} \right) = 8 \text{ hrs} \left(\frac{350 \text{ pages}}{217 \text{ pages}} \right) = 12.9 \text{ hours},$$

the same answer as with the absolute method.

18. a) 6022×10^{20}

b) 0.91×10^{-6}

c) For part (a), divide coefficient by 10^3 (or 1000), and multiply 10^{20} by 10^3 to compensate: 6.022×10^{23}

For part (b), multiply coefficient by 10^1 (or 10), and divide 10^{-6} by 10^1 to compensate: 9.1×10^{-7}

19. a) $3,300 = 3.3 \times 10^3$

b) $-3,300 = -3.3 \times 10^3$

c) $100,000,000,000 = 1.0 \times 10^{11}$, or 1×10^{11}, or just 10^{11}

d) $0.0000000048 = 4.8 \times 10^{-9}$

e) $-0.0000000048 = -4.8 \times 10^{-9}$

20. a) $9.3 \times 10^7 = 93,000,000$

b) $-9.3 \times 10^7 = -93,000,000$

c) $9.3 \times 10^{-7} = 0.00000093$

d) $10,000,000 = 1 \times 10^7$

e) $10 \times 10^7 = 100,000,000$ (ten times bigger than the previous number, so one more decimal place)

f) $10^7 = 10,000,000$ (identical to 1×10^7 in part (d) above)

g) $5.2 \times 10^0 = 5.2$

21. a) $3,000,000$

b) 3×10^6

c) twelve trillion

d) 12×10^{12} or 1.2×10^{13}

e) one hundred thousand

f) $100,000$

g) $500,000,000$ (in words, also five hundred million)
h) 0.5×10^9, or 5×10^8

i) ninety-five
j) 9.5×10^1

22. a) $(3 \times 10^5) \times 10^4 = 3 \times 10^{5+4} = 3 \times 10^9$

b) $(6 \times 10^{-6}) \times (3 \times 10^4) = (6 \times 3) \times (10^{-6+4}) = 18 \times 10^{-2}$ or, equivalently 1.8×10^{-1}, or 0.18

c) $\frac{6\times 10^{-6}}{3\times 10^4} = \frac{6}{3} \times 10^{-6-4} = 2 \times 10^{-10}$

d) $\frac{6\times 10^{6}}{1\times 10^4} = \frac{6}{1} \times 10^{6-4} = 6 \times 10^{2}$

23. a) integer rounding: $(1.23\times 10^5)\times(4.56\times 10^4) \approx (1\times 10^5)\times(5\times 10^4) = 5 \times 10^9$
R.O.M. estimation: $10^5 \times 10^5 = 10^{10}$

b) integer rounding: $\frac{9.87\times 10^{-6}}{6.54\times 10^4} \approx \frac{10\times 10^{-6}}{7\times 10^4} = \frac{10}{7} \times 10^{-6-4} = \frac{10}{7} \times 10^{-10} \approx 1 \times 10^{-10}$ (rounding coefficient)
R.O.M. estimation: $\frac{10^{-5}}{10^5} = 10^{-5-5} = 10^{-10}$ (same answer)

c) integer rounding: $\frac{(6.6\times 10^{-6})\times(2.2\times 10^4)}{(1.8\times 10^4)^2} \approx \frac{(7\times 10^{-6})\times(2\times 10^4)}{4\times 10^8} = \frac{14\times 10^{-2}}{4\times 10^8} = \frac{7}{2} \times 10^{-10} \approx 4 \times 10^{-10}$
R.O.M. estimation: $\frac{10^{-5}\times 10^4}{(10^4)^2} = \frac{10^{-1}}{10^8} = 10^{-9}$

24. a) $(2\times 10^{-4})^4 = 2^4\times(10^{-3})^4 = 16\times 10^{(-3\times 4)} = 16\times 10^{-12} = 1.6\times 10^{-11}$

b) $(4 \times 10^{14})^{-1/2} = 4^{-1/2} \times (10^{14})^{-1/2} = \frac{1}{4^{1/2}} \times 10^{(14\times\frac{-1}{2})} = \frac{1}{2} \times 10^{-7} = 0.5 \times 10^{-7} = 5 \times 10^{-8}$

c) $\sqrt[3]{1 \times 10^{-15}} = (1 \times 10^{-15})^{1/3} = 1^{1/3} \times 10^{(-15\times\frac{1}{3})} = 1 \times 10^{-5}$