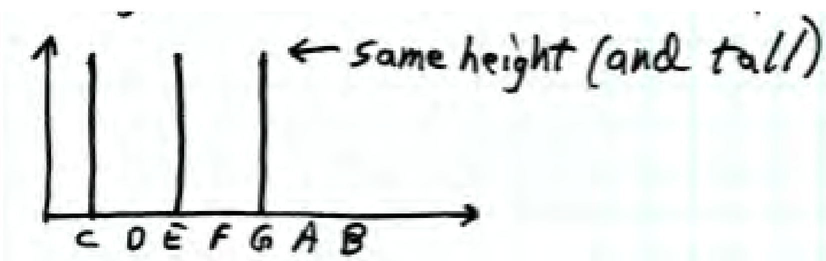
Chapter 3

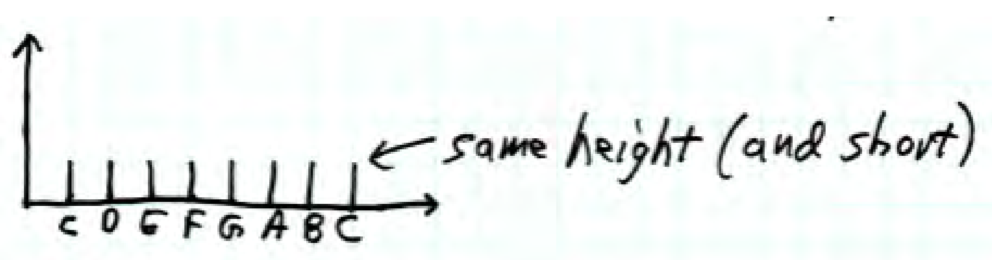
Exercise Solutions

1. Equal volume means "same height on graph."

   a) High volume means "near top of graph."



   b) Low volume means "near bottom of graph."



2. The graph in Fig. 3.3 makes it a bit hard to tell which colors have higher brightness, but we estimate this ranking:

   B = G > Y > O > R > I > IR > Radio > V > UV

3. From Eq. 3.1, $\lambda = c/f$

   $f = 3.2$ GHz $= 3.2 \times 10^9$ Hz $= 3.2 \times 10^9$ s^{-1}
   $c = 3 \times 10^8$ m/s

$$\lambda = \frac{3 \times 10^8 \text{ m/s}}{3.2 \times 10^9 \text{ s}^{-1}} = 0.094 \text{ m (ans.)}$$

4. From Eq. 3.1, $f = c/\lambda$

   $\lambda = 630$ nm $=630 \times 10^{-9}$ m $= 6.30 \times 10^{-7}$ m
   $c = 3 \times 10^8$ m/s

$$f = \frac{3 \times 10^8 \text{ m/s}}{6.30 \times 10^{-7} \text{ m}} = 4.76 \times 10^{14} \text{ Hz (ans.)}$$

5. From Eq. 3.3, $E = h/f$

   $f = 4.76 \times 10^{14}$ Hz $= f = 4.76 \times 10^{14}$ s^{-1}
   $h = 6.626 \times 10^{-34}$ J·s

$$E = hf = (6.626 \times 10^{-34}\text{Js})(4.76 \times 10^{14} \text{ s}^{-1}) = 3.15 \times 10^{-19} \text{ J (ans.)}$$

6. Since T $= \frac{0.0029 \text{ m·K}}{\lambda_{\text{peak}}}$ (Eq. 3.5), longer wavelength (peak) corresponds to lower temperature, so the curve that peaks farthest to the right represents the lowest temperature and the curve that peaks farthest to the left rpresents the highest temperature.

   So, from highest to lowest temperature, the ranking is:

   Y = Z > V > X >W (ans.)

7. If the horizontal axis represented frequency instead of wavelength, the curve with peak farthest to the right would have the highest frequency (and shortest wavelength), so the order would reverse:

   W > X > V > Z = Y

   If the horizontal axis represented energy instead of wavelength, the curve with peak farthest to the right would have the highest energy (and highest frequency and shortest wavelength), so the order would be:

   W > X > V > Z = Y

   Notice that this is the same order as for frequency, because energy is directly proportional to frequency.

8. From Eq. 3.10, $\frac{\lambda_{\text{app}}}{\lambda_{\text{true}}} = 1 + \frac{v_{\text{rec}}}{c}$

   $\lambda_{\text{app}} = 750 \text{ nm} = 750 \times 10^{-9} \text{ m} = 7.50 \times 10^{-7} \text{ m}$

   $v_{\text{rec}} = 25 \text{ km/s} = 2.5 \times 10^{4} \text{ m/s}$

   $c = 3 \times 10^{8} \text{ m/s}$

$$
\begin{aligned}
\frac{\lambda_{\text{app}}}{\lambda_{\text{true}}} &= 1 + \frac{v_{\text{rec}}}{c} \\
\lambda_{\text{app}} &= \lambda_{\text{true}} \left(1 + \frac{v_{\text{rec}}}{c}\right) \\
\lambda_{\text{true}} &= \frac{\lambda_{\text{app}}}{\left(1 + \frac{v_{\text{rec}}}{c}\right)} = \frac{7.5 \times 10^{-7} \text{ m}}{\left(1 + \frac{2.5 \times 10^{4} \text{ m/s}}{3 \times 10^{8} \text{ m/s}}\right)} \\
&= 7.499375 \times 10^{-7} \text{ m (ans.)}
\end{aligned}
$$

9. From Eq. 3.10, $\frac{\lambda_{\text{app}}}{\lambda_{\text{true}}} = 1 + \frac{v_{\text{rec}}}{c}$

   $\lambda_{\text{app}} = 1.251 \text{ cm}$

   $\lambda_{\text{true}} = 1.250 \text{ cm}$

$c = 3 \times 10^8$ m/s

$$\frac{\lambda_{\text{app}}}{\lambda_{\text{true}}} = 1 + \frac{v_{\text{rec}}}{c}$$
$$\frac{\lambda_{\text{app}}}{\lambda_{\text{true}}} - 1 = \frac{v_{\text{rec}}}{c}$$
$$v_{\text{rec}} = c\left(\frac{\lambda_{\text{app}}}{\lambda_{\text{true}}} - 1\right) = (3 \times 10^8 \text{ m/s})\left(\frac{1.251 \text{ cm}}{1.250 \text{ cm}} - 1\right) \quad = 240,000 \text{ m/s (ans.)}$$

10. Need to find f_{app}, and from Eq. 3.12, $\frac{f_{\text{true}}}{f_{\text{app}}} = 1 + \frac{v_{\text{rec}}}{c}$

$f_{\text{true}} = 18$ GHz $= 1.8 \times 10^{10}$ Hz
$v_{\text{rec}} = 150$ km/s $= 1.5 \times 10^5$ m/s
$c = 3 \times 10^8$ m/s

$$\frac{f_{\text{true}}}{f_{\text{app}}} = 1 + \frac{v_{\text{rec}}}{c}, \text{ so}$$
$$f_{\text{true}} = f_{\text{app}}\left(1 + \frac{v_{\text{rec}}}{c}\right)$$
$$f_{\text{app}} = \frac{f_{\text{true}}}{\left(1 + \frac{v_{\text{rec}}}{c}\right)} = \frac{1.8 \times 10^{10} \text{ Hz}}{\left(1 + \frac{1.5 \times 10^5 \text{ m/s}}{3 \times 10^8 \text{ m/s}}\right)}$$
$$= 1.79910045 \times 10^{10} \text{ Hz}$$

Now you can find Δf:

$$\Delta f = f_{\text{app}} - f_{\text{true}}$$
$$= 1.79910045^{10} \text{ Hz} - 1.8 \times 10^{10} \text{ Hz}$$
$$= -9.0 \times 10^6 \text{ Hz (ans.)}$$

(The change in frequency, Δf, is negative because f_{app} is smaller than f_{true}. This is always the case when either the source or observer is receding from the other.)

11. From Eq. 3.11, $\frac{\Delta\lambda}{\lambda_{\text{true}}} = \frac{v_{\text{rec}}}{c}$

$\lambda_{\text{true}} = 530.\text{ nm} = 530. \times 10^{-9}\text{ m} = 5.3 \times 10^{-7}\text{ m}$

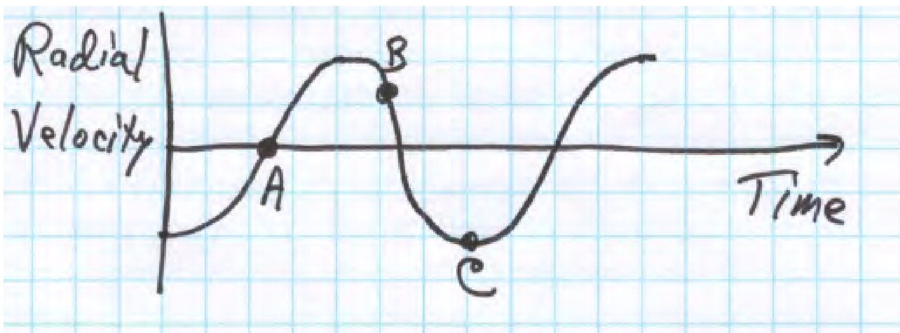
$v_{\text{rec}} =$ -300 km/s $= -3 \times 10^{5}$ m/s (negative since approaching)

$c = 3 \times 10^{8}$ m/s

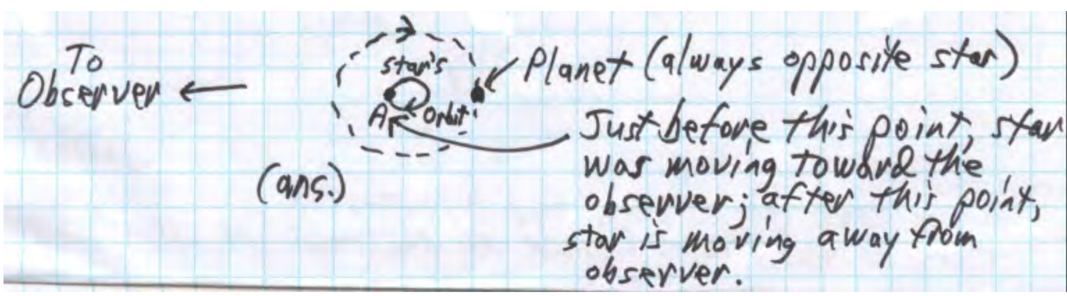
$$\begin{aligned}
\frac{\Delta\lambda}{\lambda_{\text{true}}} &= \frac{v_{\text{rec}}}{c} \\
\Delta\lambda &= \lambda_{\text{true}} \left(\frac{v_{\text{rec}}}{c}\right) = (5.3 \times 10^{-7}\text{ m}) \left(\frac{-3 \times 10^{5}\text{ m/s}}{3 \times 10^{8}\text{ m/s}}\right) \\
&= -5.3 \times 10^{-10}\text{ m (ans.)}
\end{aligned}$$

$$\begin{aligned}
\lambda_{\text{app}} - \lambda_{\text{true}} &= \Delta\lambda, \text{ so} \\
\lambda_{\text{app}} &= \lambda_{\text{true}} + \Delta\lambda \\
&= 5.3 \times 10^{-7}\text{ m} + (-5.3 \times 10^{-10}\text{ m}) \\
&= 5.2947 \times 10^{-7}\text{ m (ans.)}
\end{aligned}$$

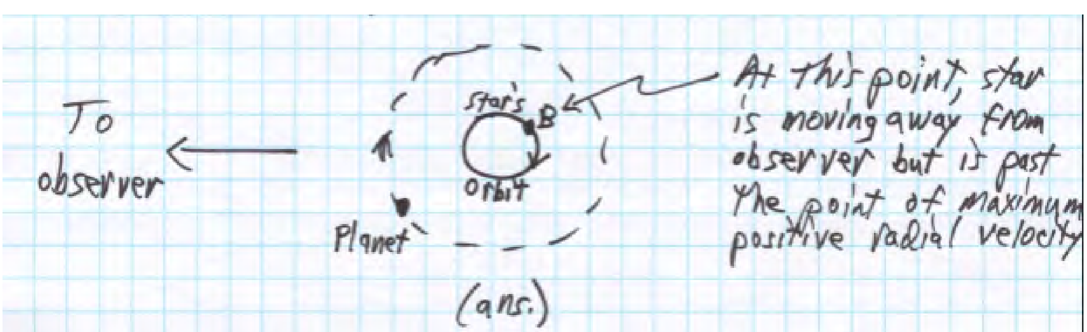
12. The figure below illustrates the points "A", "B", and "C" that are discussed in this problem:



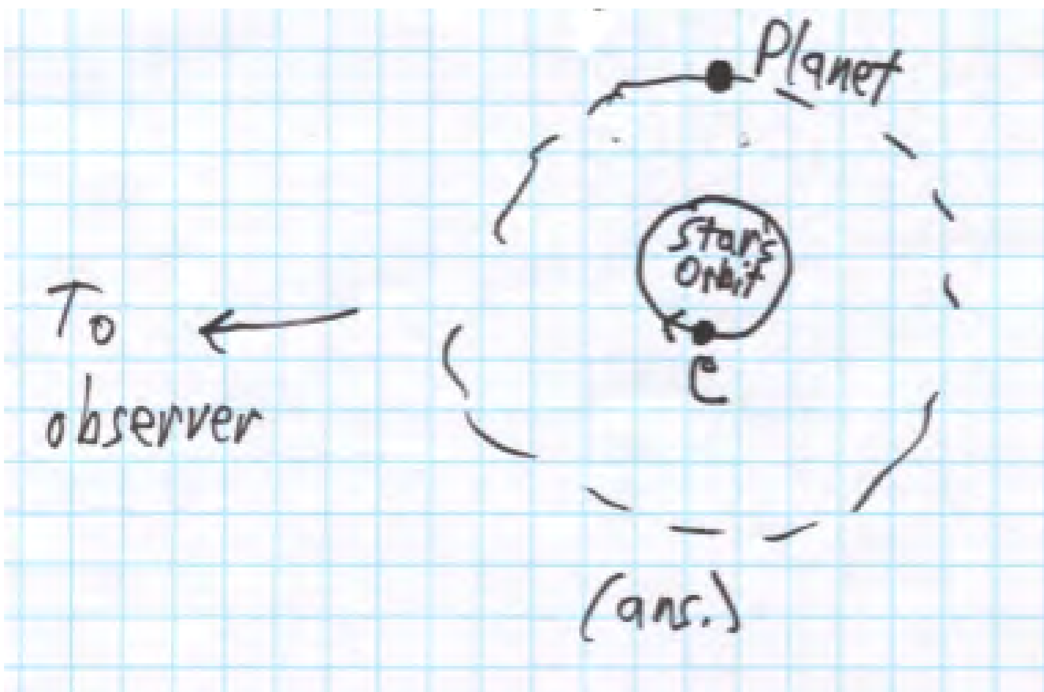
At point "A" the radial velocity is zero, so the star is moving transverse to the line of sight. And since at point "A" the radial velocity is changing from negative to positive, the star was moving toward the observer just before it reached point "A" and is moving away from the observer (positive radial velocity) just after it reached point "A". That means point "A" is at this position:



At point "B", the star is past the point of maximum positive radial velocity (moving away from observer) and is approaching the point of zero radial velocity:



At point "C", the star is at the location of maximum negative radial velocity (moving toward observer):



13. By looking at the time between the peaks in Fig. 3.18, the star's orbital period is seen to be about 20 months, and the planet's period is the same as the star's. (ans.)

14. From Kepler's Third Law (Eq. 2.12), $P^2 = a^3/M$ (as long as "P" has units of years, "a" has units of AU, and "M" has units of solar masses).

$P = 20$ months $= 1.67$ years

$M = 2.5$ solar masses

$$\begin{aligned} P^2 &= \frac{a^3}{M} \\ a^3 &= MP^2 \\ a &= \sqrt[3]{MP^2} = \sqrt[3]{(2.5)(1.67)^2} \\ &= 1.9 \text{ AU (ans.)} \end{aligned}$$

15. From Eq. 3.14, $M_{\text{actual}} = M_{\text{inferred}}/\sin i$

$M_{\text{inferred}} = 0.8\ \mathrm{M_{Jupiter}}$

$i = 45°$

$$\begin{aligned} M_{\text{actual}} &= \frac{0.8\ \mathrm{M_{Jupiter}}}{\sin 45°} \\ &= \frac{0.8\ \mathrm{M_{Jupiter}}}{\sqrt{2}/2} = \frac{0.8\ \mathrm{M_{Jupiter}}}{0.707} \\ &= 1.13\mathrm{M_{Jupiter}} \text{ (ans.)} \end{aligned}$$