Chapter 5

Exercise Solutions

1. Parallax angle $p = 0.008''$

   From Eq. 5.1, $d = 1/p$, as long as p is in units of arcseconds ($''$) and d is in units of parsecs (pc).

$$d = \frac{1}{0.008} = \frac{1}{8/1000} = \frac{1000}{8} = 125 \text{ pc}$$

$$125 \text{ pc} = 125 \cancel{\text{pc}} \times \left(\frac{3.26 \text{ ly}}{1 \cancel{\text{pc}}}\right) = 408 \text{ ly (ans.)}$$

2. From Eq. 5.1, $d = 1/p$, as long as p is in units of arcseconds ($''$) and d is in units of parsecs (pc).

$$d = \frac{1}{0.002} = \frac{1}{2/1000} = \frac{1000}{2} = 500 \text{ pc (ans.)}$$

   The smallest parallax angle corresponds to the maximum distance, since parallax angle and distance are inversely proportional.

3. From Eq. 5.1, $d = 1/p$, as long as p is in units of arcseconds ($''$) and d is in units of parsecs (pc).

   From Section 5.1, $p_{\text{Proxima}} = 100\, p_{\text{Polaris}}$, and $p_{\text{Polaris}} = 0.0075''$.

Therefore, $p_{\text{Proxima}} = 0.75''$

$$\frac{d_{\text{Kap}}}{d_{\text{Prox}}} = \frac{1/p_{\text{Kap}}}{1/p_{\text{Prox}}} = \frac{p_{\text{prox}}}{p_{\text{kap}}} = \frac{0.75''}{0.1''} = 7.5$$
$$d_{\text{Kap}} = 7.5\, d_{\text{Prox}} \text{ (ans.)}$$

Kappa Ceti is 7.5 times farther than Proxima Centauri.

4. From Eq. 5.11, apparent brightness = L / $4\pi d^2$

For Vega, $L = 40\, L_\odot = 40 \times (4 \times 10^{26} \text{ W}) = 160 \times 10^{26}$ W
d = 25 $\cancel{\text{ly}} \times \frac{9.46\times10^{15} \text{ m}}{1 \cancel{\text{ly}}} = 236.5 \times 10^{15}$ m

$$\text{apparent brightness} = \frac{160 \times 10^{26} \text{ W}}{4\pi(236.5 \times 10^{15} \text{ m})} = 2.28 \times 10^{-8} \text{ W/m}^2 \text{ (ans)}$$

Or, using ratios

$$\frac{\text{app br}_{\text{Sun}}}{\text{app br}_{\text{Vega}}} = \frac{\frac{L_{\text{Sun}}}{\cancel{4\pi} d^2_{\text{Sun}}}}{\frac{L_{\text{Vega}}}{\cancel{4\pi} d^2_{\text{Vega}}}} = \frac{L_{\text{Sun}}}{L_{\text{Vega}}}\frac{d^2_{\text{Vega}}}{d^2_{\text{Sun}}} = \frac{L_{\text{Sun}}}{L_{\text{Vega}}}\left(\frac{d_{\text{Vega}}}{d_{\text{Sun}}}\right)^2$$
$$= \frac{\cancel{L_\odot}}{40\,\cancel{L_\odot}}\left(\frac{236.5 \times 10^{15}\, \cancel{\text{m}}}{150 \times 10^{9}\, \cancel{\text{m}}}\right)^2 = 6.21 \times 10^{10} \text{ (no units)}$$
$$\text{app br}_{\text{Sun}} = (6.21 \times 10^{10})(\text{app br}_{\text{Vega}}), \text{ so}$$

the Sun appears about 62 billion times brighter than Vega. Therefore

$$\text{app br}_{\text{Vega}} = \frac{\text{app br}_{\text{Sun}}}{6.21 \times 10^{10}} = \frac{1415 \text{ W/m}^2}{6.21 \times 10^{10}} = 2.28 \times 10^{-8} \text{ W/m}^2, \text{ (ans.)}$$

in agreement with the answer above.

5. From Eq. 5.14, $\Delta m = \log_{10}$(brightness ratio) / $\log_{10}(2.512)$

Brightness ratio = 400,000 $\quad m_{\text{moon}} = -12.8$

$\Delta m = \log_{10}(400{,}000)$ / $\log_{10}(2.512)$ = 5.6 / 0.4 = 14

$m_{\text{Sun}} = m_{\text{moon}} - \Delta m = -12.8 - 14 = -26.8$ (ans.)

It makes sense that the Sun's magnitude is a more negative number, because the Sun is brighter.

6. $m = 3.2$; $d = 175$ ly

   Compare this to a star with distance = 10 pc:

   distance ratio: $\frac{10\ \cancel{\text{pc}}}{175\ \cancel{\text{ly}}} \times \frac{3.26\ \cancel{\text{ly}}}{1\ \cancel{\text{pc}}} = 0.186$

   brightness ratio: $\frac{1}{(0.186)^2} = 28.905$ (using the inverse-square law)

   From Eq. 5.14,

   $\Delta m = \frac{\log_{10}(\text{brightness ratio})}{\log_{10}(2.512)} = \frac{\log_{10}(28.905)}{\log_{10}(2.512)} = \frac{1.461}{0.4} = 3.65$

   $M = m - \Delta m = 3.2 - 3.65 = -0.45$ (ans.)

   You subtract Δm because the absolute magnitude will be *smaller* than the apparent magnitude for this star. This is because its distance (175 ly) is larger than 10 pc, so bringing it to 10 pc could make it appear brighter, which corresponds to a *smaller* magnitude.

7. $M = m - 5 \log \left[\frac{d}{10\text{ pc}}\right]$ (Eq. 5.18)

   $m = 3.2$; $d = 175\ \cancel{\text{ly}} \times \left(\frac{1\text{ pc}}{3.26\ \cancel{\text{ly}}}\right) = 53.68$ pc

   $M = 3.2 - 5 \log \left(\frac{53.68\ \cancel{\text{pc}}}{10\ \cancel{\text{pc}}}\right) = 3.2 - 5(0.7298) = -0.45$ (ans.)

   (same answer as the previous exercise)

8. a) $6 \times 10^{24}\ \cancel{\text{kg}} \times \frac{1\ \text{M}_\odot}{2\times10^{30}\ \cancel{\text{kg}}} = 3 \times 10^{-6}\ \text{M}_\odot$

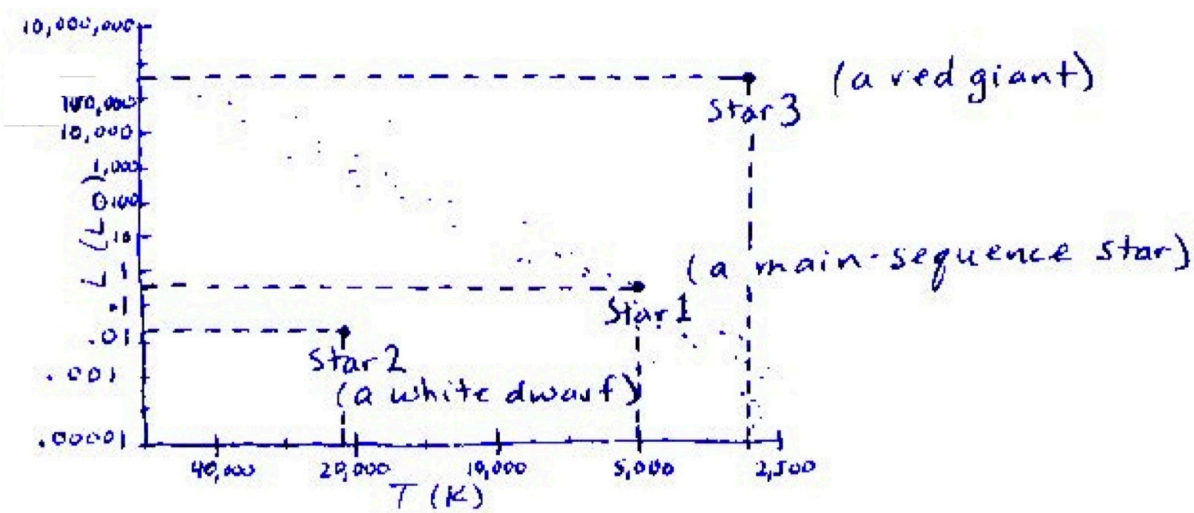
   b) $8 \times 10^{32}\ \cancel{\text{kg}} \times \frac{1\ \text{M}_\odot}{2\times10^{30}\ \cancel{\text{kg}}} = 4 \times 10^{2}\ \text{M}_\odot = 400\ \text{M}_\odot$

   c) $6378\ \cancel{\text{km}} \times \frac{1\ \text{R}_\odot}{6.96\times10^{5}\ \cancel{\text{km}}} = 0.00916\ \text{R}_\odot = 9.16 \times 10^{-3}\ \text{R}_\odot$

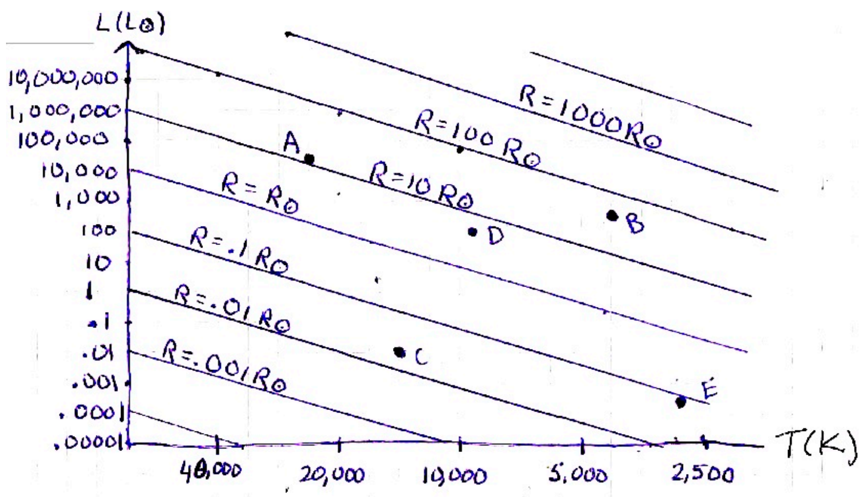
d) $40\ \cancel{\text{AU}} \times \frac{1.50\times10^{11}\ \cancel{\text{m}}}{1\ \cancel{\text{AU}}} \times \frac{1\ \text{R}_\odot}{6.96\times10^5\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{km}}}{10^3\ \cancel{\text{m}}} = 8.62 \times 10^3\ \text{R}_\odot$

e) $7 \times 10^{31}\ \cancel{\text{W}} \times \frac{1\ \text{L}_\odot}{4\times10^{26}\ \cancel{\text{W}}} = 1.75 \times 10^5\ \text{L}_\odot$

9. See figure below:



10. See figure below: Reading the star radii off the graph, in units of $R_\odot$, in-



terpolating between the labeled radius lines (diagonals), and performing a unit conversion, and rounding answers:

Star A: $10\ \cancel{R_\odot} \times \frac{6.96\times10^5\ \text{km}}{1\ \cancel{R_\odot}} = 7 \times 10^6$ km (ans.)

Star B: $80\ \cancel{R_\odot} \times \frac{6.96\times10^5\ \text{km}}{1\ \cancel{R_\odot}} = 6 \times 10^7$ km (ans.)

Star C: $0.02\ \cancel{R_\odot} \times \frac{6.96\times10^5\ \text{km}}{1\ \cancel{R_\odot}} = 1 \times 10^4$ km (ans.)

Star D: $8\ \cancel{R_\odot} \times \frac{6.96\times10^5\ \text{km}}{1\ \cancel{R_\odot}} = 6 \times 10^6$ km (ans.)

Star E: $0.09\ \cancel{R_\odot} \times \frac{6.96\times10^5\ \text{km}}{1\ \cancel{R_\odot}} = 6 \times 10^4$ km (ans.)

OR, you can use Eq. 5.19 to calculate with ratios:

Star A:

$$\frac{R_{\rm A}^2}{R_{\rm Sun}^2} = \frac{\cancel{k}\frac{L_{\rm A}}{T_{\rm A}^4}}{\cancel{k}\frac{L_{\rm Sun}}{T_{\rm Sun}^4}} = \frac{L_{\rm A}}{L_{\rm Sun}} \times \frac{T_{\rm Sun}^4}{T_{\rm A}^4}$$

$$\frac{R_{\rm A}}{R_{\rm Sun}} = \sqrt{\frac{L_{\rm A}}{L_{\rm Sun}} \times \frac{T_{\rm Sun}^4}{T_{\rm A}^4}} = \sqrt{\frac{40,000\ \cancel{L_\odot}}{1\ \cancel{L_\odot}} \times \left(\frac{5,800\ \cancel{K}}{25,000\ \cancel{K}}\right)^4} = 10.8$$

$$R_{\rm A} = 10.8\ R_{\rm Sun} = 10.8\ \mathrm{R}_\odot\ \text{(ans.)}$$

Star B:

$$\frac{R_{\rm B}}{R_{\rm Sun}} = \sqrt{\frac{L_{\rm B}}{L_{\rm Sun}} \times \frac{T_{\rm Sun}^4}{T_{\rm B}^4}} = \sqrt{\frac{500\ \cancel{L_\odot}}{1\ \cancel{L_\odot}} \times \left(\frac{5,800\ \cancel{K}}{4,000\ \cancel{K}}\right)^4} = 47$$

$$R_{\rm B} = 47\ R_{\rm Sun} = 47\ \mathrm{R}_\odot\ \text{(ans.)}$$

Star C:

$$\frac{R_{\rm C}}{R_{\rm Sun}} = \sqrt{\frac{L_{\rm C}}{L_{\rm Sun}} \times \frac{T_{\rm Sun}^4}{T_{\rm C}^4}} = \sqrt{\frac{0.01\ \cancel{L_\odot}}{1\ \cancel{L_\odot}} \times \left(\frac{5,800\ \cancel{K}}{16,000\ \cancel{K}}\right)^4} = 0.013$$

$$R_{\rm C} = 0.013\ R_{\rm Sun} = 0.013\ \mathrm{R}_\odot\ \text{(ans.)}$$

Star D:

$$\frac{R_{\rm D}}{R_{\rm Sun}} = \sqrt{\frac{L_{\rm D}}{L_{\rm Sun}} \times \frac{T_{\rm Sun}^4}{T_{\rm D}^4}} = \sqrt{\frac{100\ \cancel{L_\odot}}{1\ \cancel{L_\odot}} \times \left(\frac{5,800\ \cancel{K}}{9,000\ \cancel{K}}\right)^4} = 4.15$$

$$R_{\rm D} = 4.15\ R_{\rm Sun} = 4.15\ \mathrm{R}_\odot\ \text{(ans.)}$$

Star E:

$$\frac{R_{\rm E}}{R_{\rm Sun}} = \sqrt{\frac{L_{\rm E}}{L_{\rm Sun}} \times \frac{T_{\rm Sun}^4}{T_{\rm E}^4}} = \sqrt{\frac{0.004\ \cancel{\rm L_\odot}}{1\ \cancel{\rm L_\odot}} \times \left(\frac{5,800\ \cancel{K}}{3,000\ \cancel{K}}\right)^4} = 0.075$$

$$R_{\rm E} = 0.075\ R_{\rm Sun} = 0.075\ {\rm R}_\odot \text{ (ans.)}$$

Then, each of these radii can be converted to km using a unit conversion from $\rm R_\odot$, as previously shown.

11. From Eq. 1.12, rate = amount / time

amount $= 2.9 \times 10^{30}$ kg ; time $= 10 \times 10^6$ years

$$\text{rate} = \frac{2.9 \times 10^{30}\ \text{kg}}{10 \times 10^6\ \cancel{\text{years}}} \times \frac{1\ \cancel{\text{year}}}{3.16 \times 10^7\ \text{sec}} = 9.18 \times 10^{15}\ \text{kg/sec}$$

12. $\text{amount}_{\rm star} = \text{amount}_{\rm Sun} \times 20$

$\text{rate}_{\rm star} = \text{rate}_{\rm Sun} \times 12,000$

From Eq. 1.12, (life)time = amount / rate

$$\frac{\text{time}_{\rm star}}{\text{time}_{\rm Sun}} = \frac{\text{amount}_{\rm star}/\text{rate}_{\rm star}}{\text{amount}_{\rm Sun}/\text{rate}_{\rm Sun}} = \frac{\text{amount}_{\rm star}}{\text{amount}_\odot} \times \frac{\text{rate}_\odot}{\text{rate}_{\rm star}}$$

$$\frac{\text{time}_{\rm star}}{\text{time}_\odot} = \frac{\cancel{\text{amount}_\odot} \times 20}{\cancel{\text{amount}_\odot}} \times \frac{\cancel{\text{rate}_\odot}}{\cancel{\text{rate}_\odot} \times 12,000} = \frac{20}{12,000} = 0.00167$$

$$\text{time}_{\rm star} = 0.00167\ \text{time}_\odot = 0.00167(10^{10}\ \text{years}) = 1.67 \times 10^7\ \text{yrs (ans.)}$$

13. lifetime of star $= 10^9$ years

From Eq. 5.21, lifetime $\propto \frac{1}{M^{2.5}}$, or lifetime $= k\frac{1}{M^{2.5}}$

$$\frac{\text{lifetime}_{\rm star}}{\text{lifetime}_{\rm Sun}} = \frac{\cancel{k}\frac{1}{M_{\rm star}^{2.5}}}{\cancel{k}\frac{1}{M_{\rm Sun}^{2.5}}} = \left(\frac{M_{\rm Sun}}{M_{\rm star}}\right)^{2.5} = \left(\frac{M_{\rm Sun}}{M_{\rm star}}\right)^{\frac{5}{2}}$$

$\text{lifetime}_{\text{Sun}} = 10^{10}$ year

$M_{\text{Sun}} = 1\ \text{M}_\odot$ (by definition)

Rearrange the above expression to get masses alone. Start by raising both sides to the power 2/5, and then invert both sides:

$$\left(\frac{\text{lifetime}_{\text{star}}}{\text{lifetime}_{\text{Sun}}}\right)^{\frac{2}{5}} = \left(\left(\frac{M_{\text{Sun}}}{M_{\text{star}}}\right)^{\frac{5}{2}}\right)^{\frac{2}{5}} = \left(\frac{M_{\text{Sun}}}{M_{\text{star}}}\right)^{\frac{5}{2}\times\frac{2}{5}} = \left(\frac{M_{\text{Sun}}}{M_{\text{star}}}\right)^{1} = \frac{M_{\text{Sun}}}{M_{\text{star}}}$$

$$\left(\frac{\text{lifetime}_{\text{Sun}}}{\text{lifetime}_{\text{star}}}\right)^{\frac{2}{5}} = \frac{M_{\text{star}}}{M_{\text{Sun}}}$$

$$M_{\text{star}} = \left(\frac{\text{lifetime}_{\text{Sun}}}{\text{lifetime}_{\text{star}}}\right)^{\frac{2}{5}} M_{\text{Sun}} = \left(\frac{10^{10}\ \cancel{\text{years}}}{10^{9}\ \cancel{\text{years}}}\right)^{\frac{2}{5}} (1\ \text{M}_\odot)$$

$$M_{\text{star}} = 10^{\frac{2}{5}}\ \text{M}_\odot = 2.5\ \text{M}_\odot \text{ (ans.)}$$