Determining the mass of an extrasolar planet

As mentioned in Section 3.4, radial-velocity measurements can be combined with other information to determine the semi-major axis of an extrasolar planet's orbit and even the mass of the extrasolar planet. This document explains what "other information" is needed and how the process works.

The basis of this approach to finding the mass of an extrasolar planet is a very powerful law of physics called the conservation of momentum. The momentum of an object (usually designated as "p") is the product of its mass (m) and its velocity (v), so the defining equation for momentum is p=mv. The law of conservation of momentum says that in any closed system (that is, a system in which external forces are negligible), the total momentum of all the objects in the system cannot change. so when objects within a closed system interact with one another, the momentum of an individual object may change, but the total momentum of all of the objects within the system must remain constant.

One reason that the conservation of momentum is such a powerful law is that momentum, like velocity, is a vector quantity – that is, momentum has both magnitude and direction. But don't worry if you've never worked with vectors (and don't ever intend to), because you can understand the process of finding an extrasolar planet's math without getting into the details of the vector nature of momentum.

That's because knowing that the total momentum of all the objects in a closed system must remain the same allows you to write a very simple equation relating the magnitude of the star's momentum to the magnitude of the planet's momentum:

$$p_{Star} = p_{Planet}$$

(remember that in the usual notation lowercase "p" refers to momentum, while uppercase "P" refers to orbital period).

Since the definition of momentum is p=mv, you can write this equation as

$$m_{Star}v_{Star} = m_{Planet}v_{Planet}$$

or

$$m_{Planet} = \frac{m_{Star}v_{Star}}{v_{Planet}} \qquad (1.1)$$

Which says you can determine the mass of the extrasolar planet if you know the mass and orbital speed of the star it's orbiting as well as the speed of the planet. But radial-velocity Doppler measurements only give you the speed of the star, so how can you find the mass of the planet?

You can get a hint about that by carefully reading the text on the bottom of page 97 and the top of page 98. It says there that the "other information" needed to find the mass of an extrasolar planet comes from two sources: the H-R diagram and Kepler's Third Law as modified by Newton.

How do these additional resources come into play in this process? The role of the HR diagram is to provide the mass of the star. As you can read about in Section 5.4, a main-sequence star's position on the H-R diagram is related to its mass. So measuring a star's temperature, which tells you where the star lies on the main sequence in the H-R diagram, allows you to estimate the mass of the star.

That leaves only one unknown in Eq. 1.1: the planet's orbital speed. That's where Kerpler's Third Law comes in.

Remember that as the planet and its parent star orbit their common center of mass, the planet's orbital period must be the same as the star's. By keeping track of the star's radial velocity over time, you can see when the radial velocity returns to the same value, and that gives you the orbital period of both the star and the planet. But for Eq. 1.1 you need the planet's orbital speed, not just its orbital period. So how can you get that?

The answer is to use Kepler's Third Law as modified by Newton:

$$P^2 = \frac{a^3}{M} \qquad (1.2)$$

where P is the planet's orbital period, a is the semi-major axis of the planet's orbit, and M is the mass of the star the planet is orbiting. Since you know the planet's orbital period (P) is the same as the star's and you can get the star's mass (M) from its position on the H-R diagram, you can use Kerpler's Third Law to determine the semi-major axis (a) of the planet's orbit.

The last piece of the puzzle falls into place when you realize that knowing the semi-major axis of the planet's orbit along with the planet's orbital period allows you to estimate the planet's orbital speed. That's because for a circular (or nearly circular) orbit, the semi-major axis is the radius of the orbit, and if you know the radius, you can find the circumference just by multiplying by 2π. Since speed is just distance divided by the time it takes to cover that distance, you know that the planet's orbital speed is just the circumference of the orbit (that's the distance covered) divided by the orbital period (that's the time it takes to cover the distance):

$$v_{Planet} = \frac{2\pi a}{P} \qquad (1.3)$$

Solving Eq. 1.2 for the semi-major axis (a) and inserting this into Eq. 1.3 gives

$$v_{Planet} = \frac{2\pi\sqrt[3]{MP^2}}{P} = 2\pi\left(\frac{M}{P}\right)^{\frac{1}{3}} \qquad (1.4)$$

where M is the mass of the star being orbited and P is the star's (and planet's) orbital period.

So now you have an estimate for each of the terms on the right side of Eq. 1.1, and you can combine those terms to determine the mass of the extrasolar planet.

Here's an expanded version of Eq. 1.1:

From star's position on H-R diagram

From Doppler measurements of star's radial velocity

$$m_{Planet} = \frac{m_{Star}\, v_{Star}}{v_{Planet}}$$

From orbital period and Kepler's Third Law